## NOTE TO PRINTER

(These instructions are for the printer. They should not be duplicated.)
THIS EXAMINATION SHOULD BE PRINTED ON $8 \frac{1}{2} \times 14$ PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT OPENS LIKE A LONG BOOK.

# McGILL UNIVERSITY FACULTY OF SCIENCE SUPPLEMENTAL/DEFERRED EXAMINATION 

MATHEMATICS 189-140A
CALCULUS I
EXAMINER: Professor W. G. Brown
DATE: XXXXday, XXXXXXXX, 2001
ASSOCIATE EXAMINER: Professor D. Sussman TIME: 14:00 - 17:00 hours
FAMILY NAME:

STUDENT NUMBER: $\square$

Instructions

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing - even rough work - must be handed in.
3. Calculators are not permitted.
4. This examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9 and 10, which are blank. You are expected to show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write on the facing page. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
5. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

| $1(\mathrm{a})$/6 | 1(b) | 2(a) | 2(b) | 2(c) | 3 | 4 | 5 | $6$$/ 10$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | /6 | /6 | /6 | /6 | /10 | /10 | /10 |  |
| 7(a) | 7(b) | 7(c) | 7(d) | 8(a) | 8(b) | Total | Term |  |
| $/ 5$ | /5 | /5 | /5 | /5 | /5 | /100 | /10 |  |

1. (a) [5 MARKS] Showing all your work, evaluate $\lim _{x \rightarrow \infty}(\sqrt{x+2}-\sqrt{x})$.
(b) [5 MARKS] Showing all your work, evaluate $\lim _{x \rightarrow-1} \frac{\sin \left(x^{2}-1\right)}{x+1}$.
2. (a) [5 MARKS] Find the derivative of $\frac{x^{2}+1}{x^{2}-2}$.
(b) $[5 \mathrm{MARKS}]$ Find the derivative of $\cos \left(\frac{1}{\sqrt{x+1}}\right)$.
(c) [5 MARKS] Find the derivative of $x \sin ^{-1} x$.
3. [10 MARKS] Showing all your work, determine the greatest and least values attained by the function $f(x)=\frac{\sin x}{(7-2 \cos x)^{2}}$.
4. [10 MARKS] If $x^{3}-x y+y^{3}=1 \quad$, determine $\quad \frac{d y}{d x} \quad$ and $\quad \frac{d^{2} y}{d x^{2}} \quad$ when $x=1$ and $y=0$.
5. [10 MARKS] The illumination $I$ of an object by a light source is given by the formula $\quad I=\frac{S}{d^{2}} \quad$ units, where $S$ is the strength of the light source, and $d$ is the distance of the object from the light source. If two light sources, one 8 times as strong as the other, are placed 3 units apart, where should an object be placed on the line between them so as to receive the least illumination?
6. [10 MARKS] Use a "linear" or "tangent-line" approximation at $x=4$ to compute an approximate value for $f(x)=\sqrt{x}+\frac{1}{\sqrt{x}} \quad$ at $x=3.97$.
7. For the function $f(x)=x^{5}+x^{6}$, showing all your work
(a) [5 MARKS] Determine the intervals where $f$ is increasing, and those where it is decreasing.
(b) [5 MARKS] Determine all local maxima, and all local minima.
(c) [5 MARKS] Determine intervals of concavity, and all inflection points.
(d) [5 MARKS] Sketch a graph of the function.
8. (a) [5 MARKS] If $\quad f^{\prime}(x)=x-1 \quad$ and $\quad f(1)=\frac{1}{5} \quad$, find $f(x)$.
(b) [5 MARKS] If $\quad f^{\prime}(x)=\frac{1}{1+x^{3}} \quad$ and $f(0)=0$, use the Mean Value Theorem to show that

$$
f(2)-f(0)=\frac{2}{1+c^{3}}
$$

for some $c$ such that $0<c<2$, and deduce that $\frac{2}{9}<f(2)<2$. (Use the Mean Value Theorem - do not determine an exact formula for $f(x)$.)

## CONTINUATION PAGE FOR PROBLEM NUMBER

You must refer to this continuation page on the page where the problem is printed!

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